15/4/2020

1

① **Open Set** :- Let (X, T) be a topological space. The members of T are called T-open or simply open set

ex- $X = \{a, b, c\}$, $T = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$

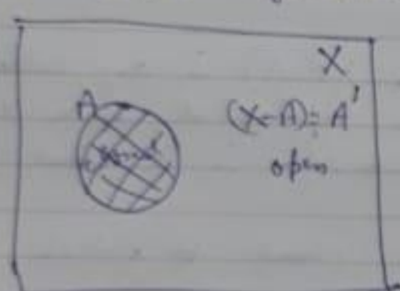
(X, T) is a topological space.

ϕ, X, $\{a\}$, $\{b\}$, $\{a, b\}$ are open set of T-open set.

② **closed set** :- (T-closed set)

Let (X, T) is a topological space. And $A \subset X$

Now 'A' is closed if A^c (A') is open.



Ex- $X = \{a, b, c\}$, $T = \{\phi, X, \{a\}, \{b\}, \{a,b\}, \{c\}, \{a,c\}, \{b,c\}\}$.

(X, T) is a topological space.

open set :- ϕ, $\{a\}$, $\{b\}$, $\{a,b\}$, $\{c\}$, $\{a,c\}$, $\{b,c\}$.

closed set :- ϕ', $\{a\}'$, $\{b\}'$, $\{a,b\}'$, $\{c\}'$, $\{a,c\}'$, $\{b,c\}'$

③ **Comparison of Topologies** :- Let T_1, T_2 are two topologies for a set X. If either $T_1 \subset T_2$ or $T_2 \subset T_1$, we say that the topologies T_1 and T_2 are compariable. If $T_1 \not\subset T_2$ and $T_2 \not\subset T_1$, then we say that T_1 and T_2 are not comparable..

Ex:- find three mutually non-comparable topologies for the set $X = \{a,b,c\}$

Sol:- (i) $T_1 = \{\phi, \{a\}, X\}$
$T_2 = \{\phi, \{b\}, X\}$
$T_3 = \{\phi, \{c\}, X\}$.

and $T_1 \not\subset T_2$, $T_1 \not\subset T_3$; $T_2 \not\subset T_3$, $T_2 \not\subset T_1$
$T_1 \not\subset T_3$, $T_1 \not\subset T_2$

(4) Intersection and Union of closed sets

If $\{F_\lambda : \lambda \in \Lambda\}$ is any collection of closed subsets of a topological space X, then $\cap\{F_\lambda : \lambda \in \Lambda\}$ is a closed set

Proof:- F_λ is closed $\forall \lambda \in \Lambda$.

$\Rightarrow F_\lambda'$ is open $\forall \lambda \in \Lambda$

$\Rightarrow \cup\{F_\lambda' : \lambda \in \Lambda\}$ is open

$\Rightarrow \{\cap\{F_\lambda : \lambda \in \Lambda\}\}'$ is open (De Morgan Law)

$\Rightarrow \cap\{F_\lambda : \lambda \in \Lambda\}$ is closed (Def. of closed set)